Flow Routing Methods

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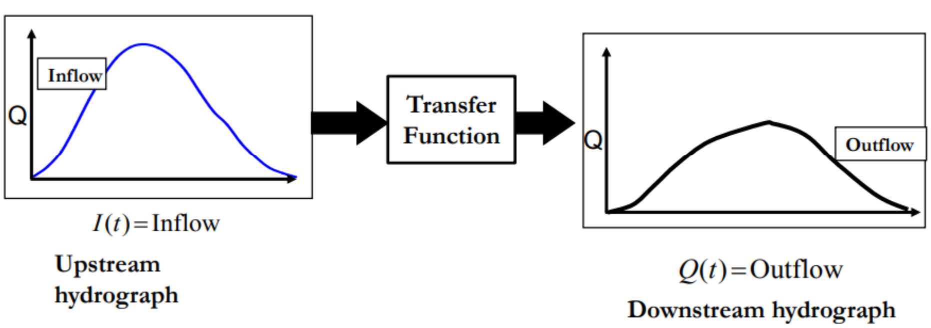
What is FLOW ROUTING?

✓ "Flow routing is a technique of determining the flow hydrograph at a section of a river by utilizing the data of flood flow at one or more upstream sections."

(Subramanya, 1984)



Types of flow routing

Lumped/hydrologic:

- **Flow ⟶ f(time)**
- **Continuity equation and Flow/Storage relationship**

Distributed/hydraulic:

- **Flow ⟶ f(space, time)**
- **Continuity and Momentum equations**

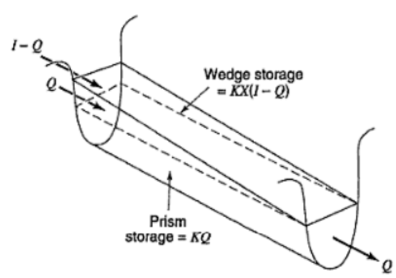
Hydrologic flow routing

Channel Routing

The total volume in storage for a channel reach having a flood wave can be considered as ***prism storage + wedge storage***.

Prism storage: The volume that would exist if uniform flow occurred at the downstream depth i.e. the volume formed by an imaginary plane parallel to the channel bottom drawn at the outflow section water surface.

Wedge storage: It is the wedge like volume formed between the actual water surface profile and the top surface of the prism storage.



Prism and wedge storage in a channel reach (Mays, 2009)

Hydrologic river routing (Muskingum Method)

Wedge storage in reach

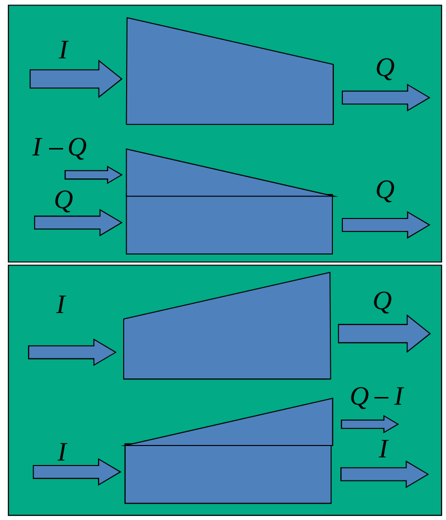
$$S_{Prism} = KQ$$

$$S_{Wedge} = KX(I - Q)$$

K = travel time of peak through the reach
X = weight on inflow versus outflow (0 ≤ X ≤ 0.5)
X = 0 ➔ Reservoir, storage depends on outflow, no wedge
X = 0.0 - 0.3 ➔ Natural stream

$$S = KQ + KX(I - Q)$$

$$S = K[XI + (1 - X)Q]$$



Muskingum Method (Cont.)

$$S = K[XI + (1 - X)Q]$$

$$S_{j+1} - S_j = K\{[XI_{j+1} + (1 - X)Q_{j+1}] - [XI_j + (1 - X)Q_j]\}$$

Recall:

$$S_{j+1} - S_j = \frac{I_{j+1} + I_j}{2}\Delta t - \frac{Q_{j+1} + Q_j}{2}\Delta t$$

Combine:

$$Q_{j+1} = C_1 I_{j+1} + C_2 I_j + C_3 Q_j$$

$$C_1 = \frac{\Delta t - 2KX}{2K(1 - X) + \Delta t}$$

$$C_2 = \frac{\Delta t + 2KX}{2K(1 - X) + \Delta t}$$

$$C_3 = \frac{2K(1 - X) - \Delta t}{2K(1 - X) + \Delta t}$$

If *I(t)*, *K* and *X* are known, *Q(t)* can be calculated using above equations

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Muskingum - Example

- Given:
  - Inflow hydrograph
  - K = 2.3 hr, X = 0.15, Δt = 1 hour, Initial Q = 85 cfs
- Find:
  - Outflow hydrograph using Muskingum routing method

$$C_1 = \frac{\Delta t - 2KX}{2K(1-X)+\Delta t} = \frac{1-2*2.3*0.15}{2*2.3(1-0.15)+1} = 0.0631$$

$$C_2 = \frac{\Delta t + 2KX}{2K(1-X)+\Delta t} = \frac{1+2*2.3*0.15}{2*2.3(1-0.15)+1} = 0.3442$$

$$C_3 = \frac{2K(1-X)-\Delta t}{2K(1-X)+\Delta t} = \frac{2*2.3*(1-0.15)-1}{2*2.3(1-0.15)+1} = 0.5927$$

Period (hr)	Inflow (cfs)
1	93
2	137
3	208
4	320
5	442
6	546
7	630
8	678
9	691
10	675
11	634
12	571
13	477
14	390
15	329
16	247
17	184
18	134
19	108
20	90

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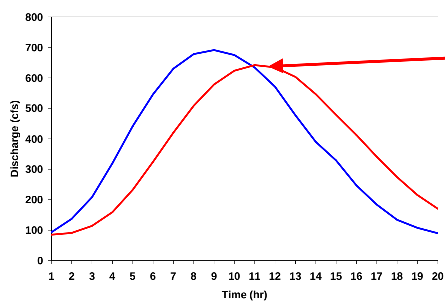
Muskingum – Example (Cont.)

$$Q_{j+1} = C_1 I_{j+1} + C_2 I_j + C_3 Q_j$$

C_1 = 0.0631, C_2 = 0.3442, C_3 = 0.5927



Period (hr)	Inflow (cfs)	C_1I_{j+1}	C_2I_j	C_3Q_j	Outflow (cfs)
1	93	0	0	0	85
2	137	9	32	50	91
3	208	13	47	54	114
4	320	20	72	68	159
5	442	28	110	95	233
6	546	34	152	138	324
7	630	40	188	192	420
8	678	43	217	249	509
9	691	44	233	301	578
10	675	43	238	343	623
11	634	40	232	369	642
12	571	36	218	380	635
13	477	30	197	376	603
14	390	25	164	357	546
15	329	21	134	324	479
16	247	16	113	284	413
17	184	12	85	245	341
18	134	8	63	202	274
19	108	7	46	162	215
20	90	6	37	128	170

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EXAMPLE 8.5 *Route the following flood hydrograph through a river reach for which K = 12.0 h and x = 0.20. At the start of the inflow flood, the outflow discharge is 10 m^3/s.*

Time (h)	0	6	12	18	24	30	36	42	48	54
Inflow (m^3/s)	10	20	50	60	55	45	35	27	20	15

SOLUTION: Since K = 12 h and $2Kx = 2 \times 12 \times 0.2 = 4.8$ h, Δt should be such that 12 h > Δt > 4.8 h. In the present case Δt = 6 h is selected to suit the given inflow hydrograph ordinate interval.

Using Eqs. (8. 16-a, b & c) the coefficients C_0, C_1 and C_2 are calculated as

$$C_0 = \frac{-12 \times 0.20 + 0.5 \times 6}{12 - 12 \times 0.2 + 0.5 \times 6} = \frac{0.6}{12.6} = 0.048$$

$$C_1 = \frac{12 \times 0.2 + 0.5 \times 6}{12.6} = 0.429$$

$$C_2 = \frac{12 - 12 \times 0.2 - 0.5 \times 6}{12.6} = 0.523$$

For the first time interval, 0 to 6 h,

$I_1 = 10.0$ $\quad$ $C_1 I_1 = 4.29$

$I_2 = 20.0$ $\quad$ $C_0 I_2 = 0.96$

$Q_1 = 10.0$ $\quad$ $C_2 Q_1 = 5.23$

From Eq. (8.16) $\quad Q_2 = C_0 I_2 + C_1 I_1 + C_2 Q_1 \quad = 10.48\ m^3/s$

For the next time step, 6 to 12 h, Q_1 = 10.48 m^3/s. The procedure is repeated for the entire duration of the inflow hydrograph. The computations are done in a tabular form as shown in Table 8.4. By plotting the inflow and outflow hydrographs the attenuation and peak lag are found to be 10 m^3/s and 12 h respectively.

Table 8.4 Muskingum Method of Routing—Example 8.5

Δt = 6 h

Time (h)	I (m^3/s)	0.048 I_2	0.429 I_1	0.523 Q_1	Q (m^3/s)
1	2	3	4	5	6
0	10				10.00
		0.96	4.29	5.23	
6	20				10.48
		2.40	8.58	5.48	
12	50				16.46
		2.88	21.45	8.61	
18	60				32.94
		2.64	25.74	17.23	
24	55				45.61
		2.16	23.60	23.85	
30	45				49.61
		1.68	19.30	25.95	
36	35				46.93
		1.30	15.02	24.55	
42	27				40.87
		0.96	11.58	21.38	
48	20				33.92
		0.72	8.58	17.74	
54	15				27.04

Hydrologic flow routing

Modified Pul's Method

$$I - Q = \frac{dS}{dt}$$

$$\bar{I}\,\Delta t - \bar{Q}\,\Delta t = \Delta S$$

$$\left(\frac{I_1 + I_2}{2}\right)\Delta t - \left(\frac{Q_1 + Q_2}{2}\right)\Delta t = S_2 - S_1$$

$$\left(\frac{I_1 + I_2}{2}\right)\Delta t + \left(S_1 - \frac{Q_1\,\Delta t}{2}\right) = \left(S_2 + \frac{Q_2\,\Delta t}{2}\right)$$

Flow routing in channels

- Distributed Routing
- St. Venant equations
  - Continuity equation

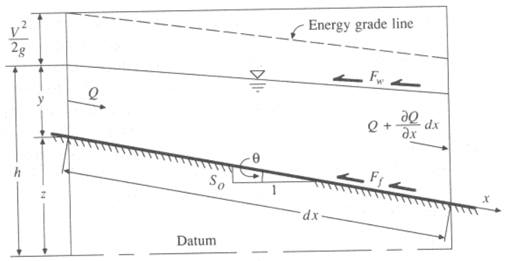
$$\frac{\partial Q}{\partial x} + \frac{\partial A}{\partial t} = 0$$

  - Momentum Equation

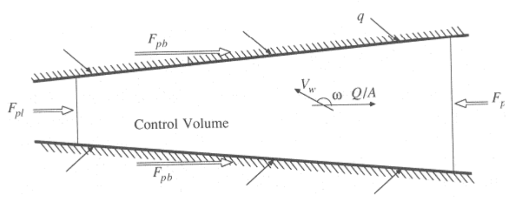
$$\frac{1}{A}\frac{\partial Q}{\partial t} + \frac{1}{A}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + g\frac{\partial y}{\partial x} - g(S_o - S_f) = 0$$

What are all these terms, and where are they coming from?

Continuity Equation



Elevation View



Plan View

Q = inflow to the control volume

q = lateral inflow

$\frac{\partial Q}{\partial x}$ Rate of change of flow with distance

$Q + \frac{\partial Q}{\partial x}dx$ Outflow from the C.V.

$\frac{\partial(\rho A dx)}{\partial t}$ Change in mass

Reynolds transport theorem

$$0 = \frac{d}{dt}\iiint_{c.v.} \rho d\forall + \iint_{c.s.} \rho V.dA$$

Momentum Equation

- From Newton's 2nd Law:
- Net force = time rate of change of momentum

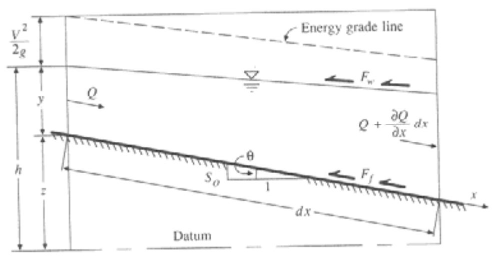
$$\sum F = \frac{d}{dt}\iiint_{c.v.} V\rho d\forall + \iint_{c.s.} V\rho V.dA$$

Sum of forces on the C.V.

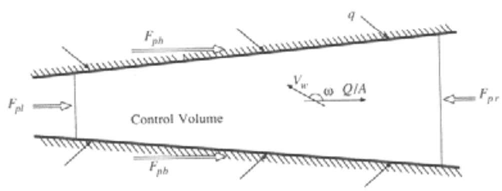
Momentum stored within the C.V

Momentum flow across the C. S.

Forces acting on the C.V.



Elevation View



Plan View

- F_g = Gravity force due to weight of water in the C.V.
- F_f = friction force due to shear stress along the bottom and sides of the C.V.
- F_e = contraction/expansion force due to abrupt changes in the channel cross-section
- F_w = wind shear force due to frictional resistance of wind at the water surface
- F_p = unbalanced pressure forces due to hydrostatic forces on the left and right hand side of the C.V. and pressure force exerted by banks

Momentum Equation

$$\sum F = \frac{d}{dt}\iiint_{c.v.} V\rho d\forall + \iint_{c.s.} V\rho V.dA$$

Sum of forces on the C.V.

Momentum stored within the C.V

Momentum flow across the C. S.

$$\frac{1}{A}\frac{\partial Q}{\partial t} + \frac{1}{A}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right) + g\frac{\partial y}{\partial x} - g(S_o - S_f) = 0$$

$$\frac{1}{A}\frac{\partial Q}{\partial t}+\frac{1}{A}\frac{\partial}{\partial x}\left(\frac{Q^2}{A}\right)+g\frac{\partial y}{\partial x}-g(S_o-S_f)=0$$

Local acceleration term	Convective acceleration term	Pressure force term	Gravity force term	Friction force term

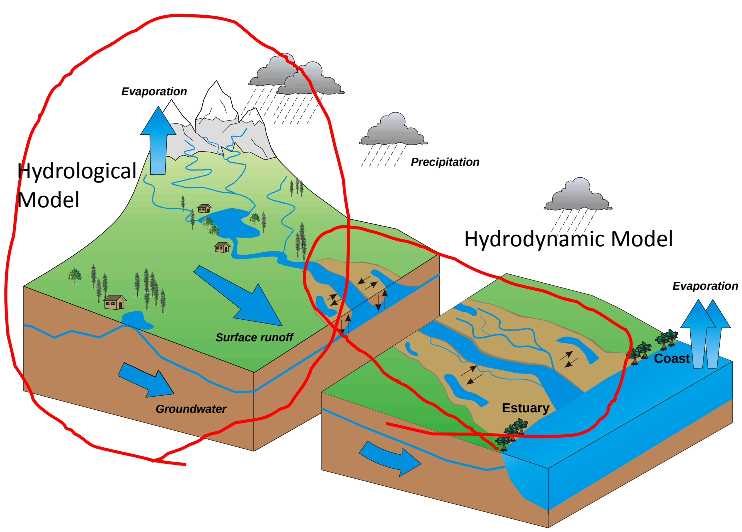
$$\frac{\partial V}{\partial t}+V\frac{\partial V}{\partial x}+g\frac{\partial y}{\partial x}-g(S_o-S_f)=0$$

Kinematic Wave

Diffusion Wave

Dynamic Wave

Flood Modeling Processes that can be simulated

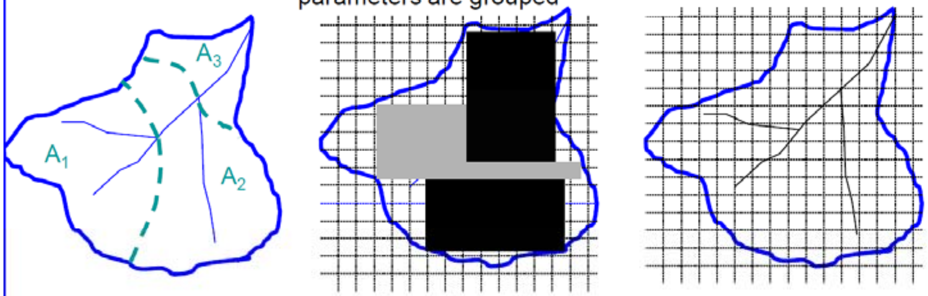


Hydrological Model Classification

Lumped
Parameters assigned to each sub-basin

Semi-Distributed
Parameters assigned to each grid cell, but cells with same parameters are grouped

Fully-Distributed
Parameters assigned to each grid cell



BROAD METHODOLOGY

Stages in the Flood Forecasting

- Computing runoff volume
  - SCS Curve Number Loss
- Modelling direct runoff
  - SCS Transform Method
- Flood Routing
  - Muskingum
- Calibration of the model
- Model validation

Spatial and non-spatial Database

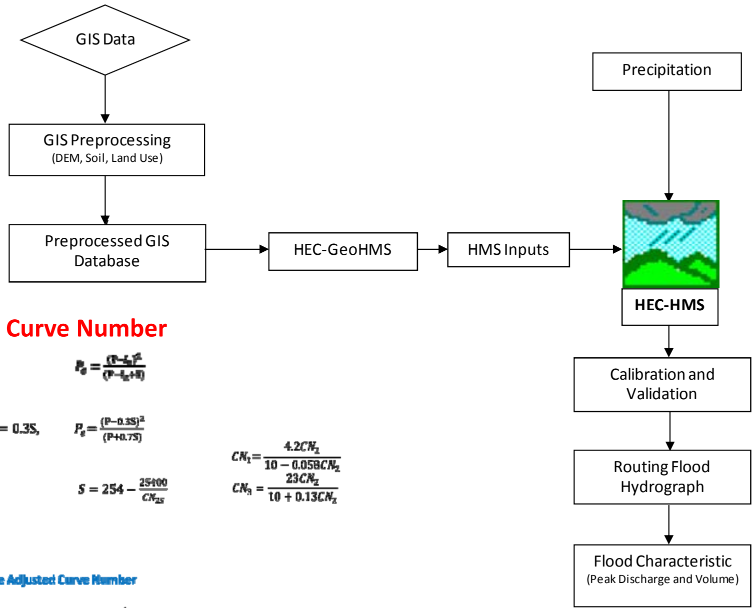
Static Data Used

Land use, Soil Texture, CARTO DEM

Dynamic Data

- Historic Hydro-meteorological Data
- Real-time 3 hr. hydrological data (CWC)
- Real time In-situ 24 hr Rainfall Data (IMD)
- 24 hr Rainfall Forecast Data at 3 hr frequency (9 km grids from IMD)
- Monthly ET data (computed)

HEC HMS (Curve Number Method)



SCS Curve Number

$$P_e = \frac{(P-I_a)^2}{(P-I_a+S)}$$

If $I_a = 0.3S$, $P_e = \frac{(P-0.3S)^2}{(P+0.7S)}$

$$CN_1 = \frac{4.2CN_2}{10-0.058CN_2}$$

$$S = 254 - \frac{25400}{CN_{2S}} \qquad CN_3 = \frac{23CN_2}{10+0.13CN_2}$$

Slope Adjusted Curve Number

$$CN_{2s} = \frac{1}{3}(CN_3 - CN_2)[1 - 2e^{-13.86Slope}] + CN_2$$

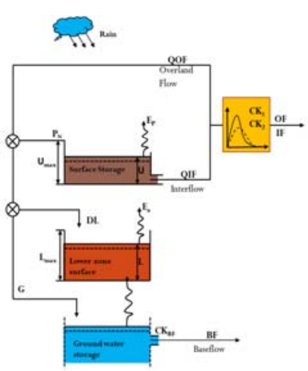
MIKE 11 (NAM (Nedbør-Afstrømings-Model))

Input Data:

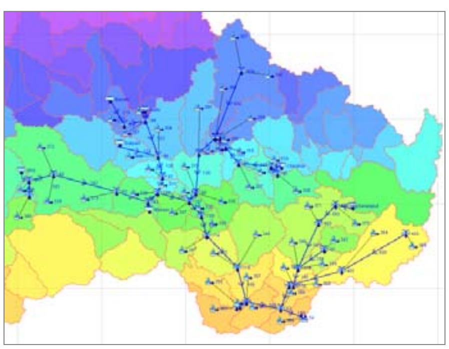
Data Requirement
• Meteorological data (rainfall and potential evaporation)
• Hydrological data (discharge at the outlet of the catchments for model calibration and validation)
• Model parameters (time constants and threshold values for routing surface storage, rootzone storage and groundwater storage)

Model Parameter:

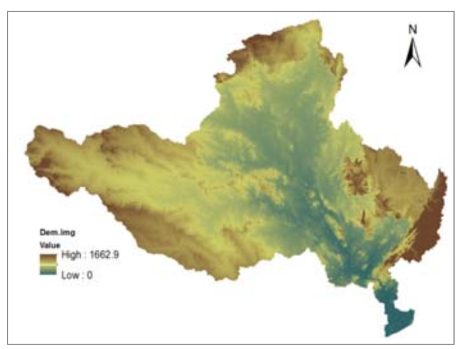
Parameter	Description	Unit
Umax (mm)	Maximum water content in surface storage	mm
Lmax (mm)	Maximum water content in root zone storage	mm
CQOF	Overland flow runoff coefficient	-
CKIF (h)	Time constant for interflow	h
TOF	Root zone threshold value for overland flow	-
CK1,2 (h)	Time constant for routing overland flow	H
TIF	Root zone threshold value for interflow	-
TG	Root zone threshold value for groundwater recharge	-
CKBF (h)	Time constant for routing base flow	h



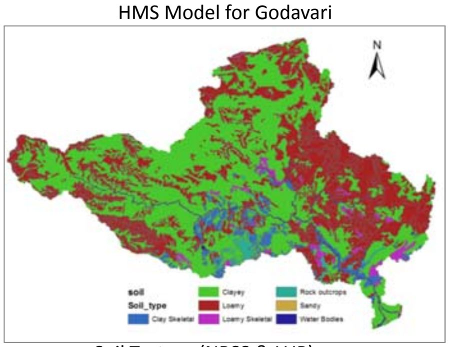
Case Study of Godavari



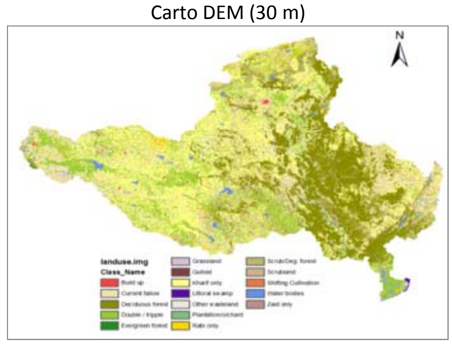
HMS Model for Godavari



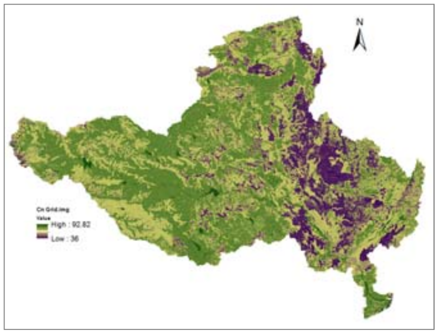
Carto DEM (30 m)



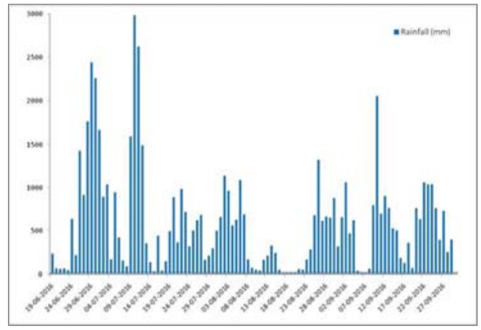
Soil Texture (NBSS & LUP)



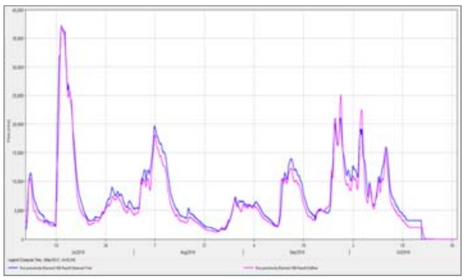
LULC (AWIFS)



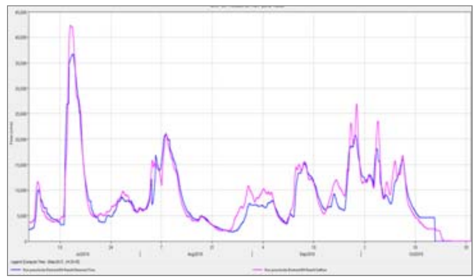
Curve Number Grid



Rainfall Pattern for Godavari (june-sep2016)

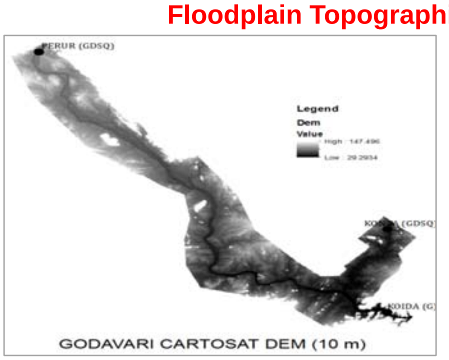


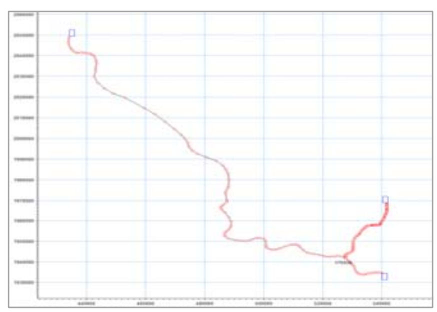
Discharge Hydrograph at Perur



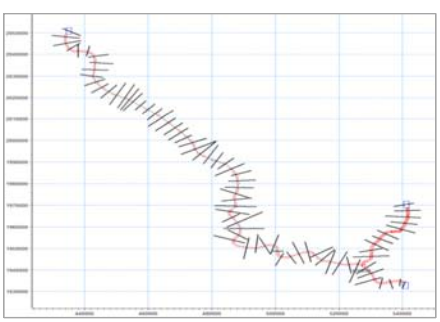
Discharge Hydrograph at Koida

Floodplain Topographic Parameter Extraction

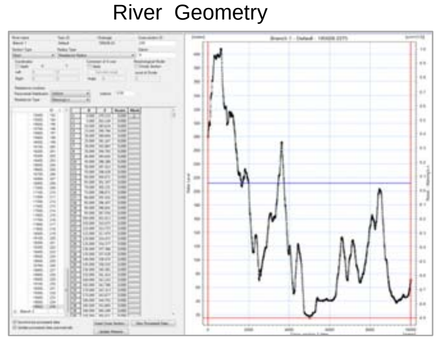




River Geometry

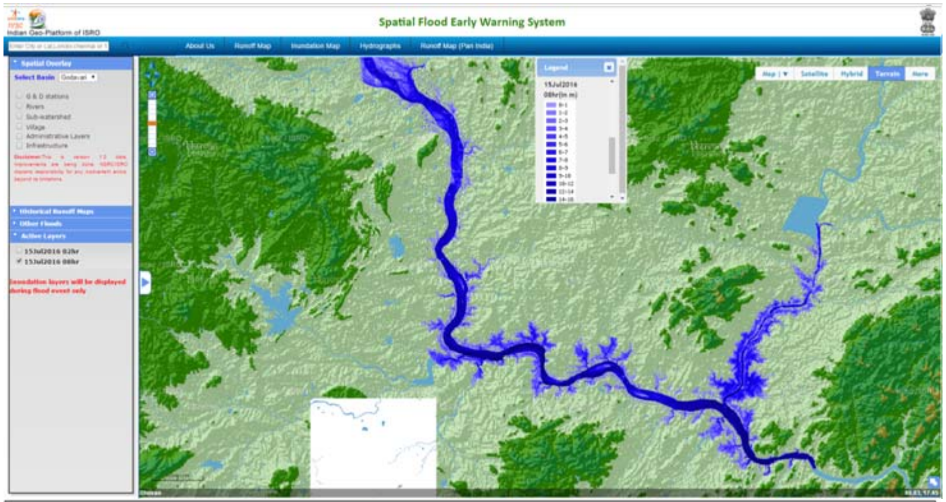


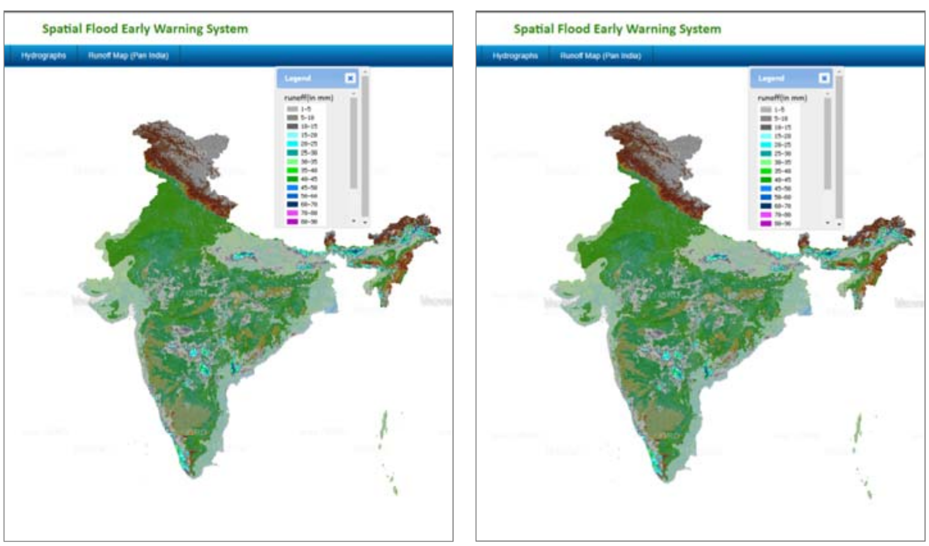
Cross- Sections



Cross- Section Profile

Web-enabled Spatial Flood Early Warning System for the Godavari Basin





Spatial Runoff Pattern

References

Engineering Hydrology. –Ojha—Oxford University Press 6. Engineering hydrology – K. Subramanyam Tata McGraw Hill, 2009.

Applied Hydrology – Chow, Maidment, Mays, McGraw-Hill

Mays, L.W., Water Resources Engineering, John Willey and Sons, US, 2010.

Thank You